Optimal trajectory generation for polynomial nonlinear systems with guaranteed constraint satisfaction using B-splines

Wannes Van Loock, Goele Pipeleers and Jan Swevers
Department of Mechanical Engineering, KU Leuven, Celestijnenlaan 300B, 3001 Heverlee
Email: wannes.vanloock@mech.kuleuven.be

1 Introduction

This contribution extends the results from [1], in which optimal trajectories for linear systems are determined using a B-spline parameterization of the flat output of the system. Semi-infinite bounds on system states or inputs are imposed using the convex hull property of B-splines, yielding simple linear constraints on the spline coefficients which guarantee constraint satisfaction over the support of the spline. This method provides an alternative to gridding the constraint, which is commonly used for solving optimal control problems but does not provide any guarantees for feasibility in between grid points.

2 Methodology

In this work an extension towards polynomial nonlinear systems is developed. Consider a nonlinear system with *polynomial* dependence on parameters p that can be written in the following control canonical state space form:

$$\dot{x} = \begin{pmatrix} 0 & & I_{n-1} & \\ -a_0(p) & -a_1(p) & \cdots & -a_{n-1}(p) \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u.$$

Now consider parameterizing both the flat output x_1 of this system and the evolution of parameters as B-splines:

$$x_1 = \sum_i c_i b_i(t), \; p_j = \sum_i d_{ij} b_{ij}(t).$$

Each of the system states and input can now be written as a polynomial function of the time-varying parameters p and the flat output x_1 and its derivatives x_2 to x_n. This implicates that all system states and inputs can also be written as a B-spline whose coefficients $\tilde{c}$ depend polynomially on those of the flat output and the time-varying parameters:

$$u = \sum_i \tilde{c}_i \tilde{b}_i(t) \text{ with } \tilde{c} = f(c,d).$$

The convex hull property of B-splines, which states that a B-spline is always contained in its control polygon defined by the coefficients, provides an elegant way of imposing semi-infinite constraints on states or inputs. Optimizing for both the states and parameters results in a non-convex optimization problem due to the polynomial dependence. However, it can still be solved efficiently using interior point algorithms such as IPOPT.

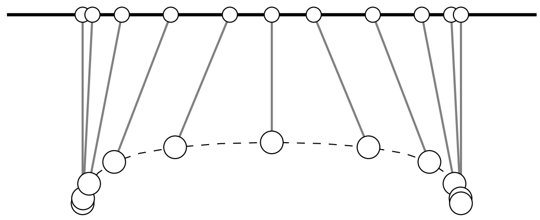

Figure 1: Time-optimal movement of an overhead crane with varying pendulum length.

3 Illustration

An application example on an overhead crane with varying pendulum length shows the potential of the proposed approach. Figure 1 illustrates a time-optimal movement of the crane with equal starting and ending pendulum lengths. The pendulum is constrained to start and stop at rest which eliminates residual vibrations. The figure shows it is beneficial to reduce the pendulum length during movement. For the considered example, an improvement of 6 % in time-optimality is achieved compared to a fixed-length pendulum.

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References

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